

Coulomb Effect on $\phi \rightarrow K^+K^-$ Invariant Mass

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Abstract

We demonstrate that coulomb interaction between a static charge source and K^+ and K^- from ϕ meson decay could introduce systematic shift in the ϕ invariant mass reconstructed from the K^+K^- pair.

1 Introduction

Hot and dense matter is created in heavy ion collisions at the BNL AGS and the CERN SPS. It is predicted that characteristics of hadrons, such as their masses and widths, can be modified in hot and/or dense matter [1]. Such modifications are of importance because they signal chiral symmetry restoration predicted by QCD [2].

The ϕ meson is an ideal candidate to look for such modifications for the following reasons. First, ϕ mesons are short-lived, therefore relatively large fraction of ϕ 's decay inside the charge fireball created in heavy ion collision; meanwhile, the ϕ mass peak is still narrow enough to be sensitive to small shift in the ϕ mass. Second, the decay momentum of $\phi \rightarrow K^+K^-$ is very small, therefore momenta of the secondary kaons do not contribute significantly to the invariant mass, resulting in little sensibility of any experimental uncertainty in the kaon momentum measurement on the ϕ mass.

The ϕ meson was measured by the BNL AGS experiment 859 in Si+Au collisions at 14.6 A·GeV/c [3]. An analysis of the data showed that the ϕ invariant mass from K^+K^- pairs detected in the E859 spectrometer decreased systematically with increasing centrality, reaching a value about 2 MeV smaller than the data-book ϕ mass in the most central events [4]. The decrease was even more pronounced for ϕ 's

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at low p_\perp . No experimental systematics have been identified so far to be responsible for the decrease in the invariant mass.

In this paper, we will demonstrate that electro-magnetic coulomb interaction between the positive charge fireball and the K^+ and K^- from a ϕ decay could give rise of systematic shift in the ϕ invariant mass, which is on the same order of that observed in the experiment. We will first give the formalism in Sect. 2, and then describe in Sect. 3 the *Monte Carlo* procedure we used implementing the experimental acceptance. We will present the results of our calculation in Sect. 4, followed by discussion of limitations of the analysis. In Sect. 6, we will give our prediction of ϕ mass shift due to coulomb effect in central Au+Au collision, and finally we will conclude in Sect. 7.

2 Relativistic Analysis

Suppose a static, spherical and uniform charge fireball is created in central heavy ion collision. (We will present all the formula in this section in the rest frame of the fireball.) The coulomb potential produced by the fireball at a distance r from origin of the fireball is given by

$$V(r) = \begin{cases} \frac{Z\alpha}{r} & (r \geq R) \\ \frac{Z\alpha}{2R} \left[3 - \left(\frac{r}{R} \right)^2 \right] & (r < R) \end{cases} \quad (1)$$

for positive unit charge particles, where Z and R are the charge and the radius of the fireball, and $\alpha = \frac{1}{137}$ is the electro-magnetic coupling constant.

The coulomb force will put a \pm impulse onto the K^\pm momentum. Due to the momentum impulses, there will be a shift in the ϕ invariant mass, the square of which is given by

$$m_\phi^2 = (E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2, \quad (2)$$

where E_+ , \vec{p}_+ and E_- , \vec{p}_- are energy and momentum-vector of the K^+ and K^- , respectively. Thus, we can obtain the ϕ mass shift,

$$\Delta m_\phi = \frac{1}{m_\phi} \left[(E_- - E_+) V(r) - V^2(r) + p_+ p_- \cos \theta_i - \right.$$

$$\sqrt{p_+^2 + 2E_+V(r) + V^2(r)} \sqrt{p_-^2 - 2E_-V(r) + V^2(r)} \cos \theta_f \Big], \quad (3)$$

where θ_i and θ_f are the opening angles at the decay point and at ∞ where the kaons are measured. Expanding Eq. 3 to the first order of $V(r)$ and assuming that the opening angle does not change and the kaon momenta are large, we obtain

$$\Delta m_\phi \approx \frac{V(r)}{m_\phi} \times \left\{ (p_- - p_+) \left(1 - \frac{m_K^2}{2p_+p_-} \right) - \left(\frac{p_-}{\beta_+} - \frac{p_+}{\beta_-} \right) \left[1 + \frac{m_K^2}{2p_+p_-} \left(\frac{p_+}{p_-} + \frac{p_-}{p_+} + 2 \right) - \frac{m_\phi^2}{2p_+p_-} \right] \right\} \quad (4)$$

where $\beta_\pm = p_\pm/E_\pm$ and m_K is the rest mass of kaon. As seen from Eq. 4, the mass shift vanishes on the first order when the kaon momenta are equal, and is negative for $p_- > p_+$ and positive for $p_+ > p_-$. However, the second order effect is always positive. Therefore, averaging over all possibilities of the ϕ decay kinematics, the invariant mass will have slightly positive shift.

Consider a central Si+Au collision. A fireball with positive net charge consisting of almost all protons from the Si projectile and a considerable fraction of protons from the Au target is formed. Using the simple spectator-participant geometry model of heavy ion collision, we can easily calculate the number of target participant nucleons, hence the amount of charge in the fireball, $Z = 43$. Assuming that the nuclear density of the fireball at freeze-out is as same as the normal nuclear density, we obtain the radius of the fireball, $R = 5.2$ fm. Suppose the ϕ decays at origin and neglect change in the opening angle, then we can calculate the mass shift according to Eq. 3 as a function of p_+ and p_- . The result is shown in Fig. 1, where the mass shift of each contour is indicated by the number beside. As seen from the figure, the mass shift is not symmetric around $p_+ = p_-$ due to the second order effect.

The opening angle could change if the ϕ decays at a distance from the origin. The angles between the K^\pm momentum and the ϕ momentum, θ_\pm , are given by

$$\sin^2 \theta_\pm = \frac{\sin^2 \theta_i}{1 + 2 \left(\frac{p_\pm}{p_\mp} \right) \cos \theta_i + \left(\frac{p_\pm}{p_\mp} \right)^2}. \quad (5)$$

Assume the kaon momenta are not small, so that we can approximate the kaon trajectories as straight lines as the kaons escape from the fireball. Using the coulomb

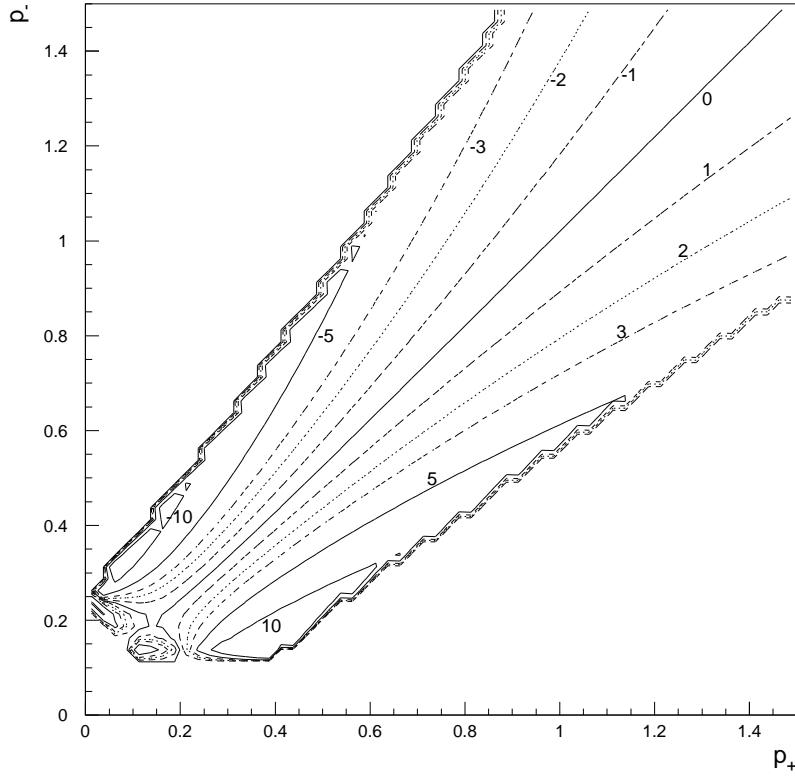


Figure 1: Shift in the invariant mass of $\phi \rightarrow K^+K^-$ versus p_+ and p_- , the K^+ and K^- momenta in GeV/c . Numbers indicated by the contours are the corresponding ϕ mass shifts in MeV/c^2 .

force derived from Eq. 1,

$$\vec{E}(r) = \begin{cases} Z\alpha \cdot \vec{r}/r^3 & (r \geq R) \\ Z\alpha \cdot \vec{r}/R^3 & (r < R) \end{cases}, \quad (6)$$

we obtain the K^\pm momentum change in the direction perpendicular to the trajectory for ϕ decay outside the spherical fireball as

$$\Delta p_\pm(r)^{out} \approx \mp Z\alpha \cdot \frac{b_\pm}{\beta_\pm} \int_{\sqrt{r^2-b_\pm^2}}^{\infty} (x^2 + b_\pm^2)^{-3/2} dx = \mp \frac{Z\alpha}{\beta_\pm r} \cdot \frac{1 - \cos \theta_\pm}{\sin \theta_\pm}, \quad (7)$$

and for ϕ decay inside as

$$\begin{aligned} \Delta p_\pm(r)^{in} &\approx \Delta p_\pm(R)^{out} \mp Z\alpha \cdot \frac{b_\pm}{\beta_\pm R^3} \int_{\sqrt{r^2-b_\pm^2}}^{\sqrt{R^2-b_\pm^2}} dx \\ &= \Delta p_\pm(R)^{out} \mp \frac{Z\alpha}{\beta_\pm R} \cdot \frac{r \sin \theta_\pm}{R} \left[\sqrt{1 - \left(\frac{r}{R}\right)^2 \sin^2 \theta_\pm} - \frac{r}{R} \cos \theta_\pm \right], \end{aligned} \quad (8)$$

where $b_{\pm} = r \sin \theta_{\pm}$ is the distance between the K^{\pm} straight line trajectory and the origin of the fireball. Thus, the angle deflection can be obtained by

$$\Delta\theta_{\pm} = \arcsin \frac{\Delta p_{\pm}}{p_{\pm}}, \quad (9)$$

and therefore the change in the opening angle by

$$\Delta\theta \equiv \theta_f - \theta_i = \Delta\theta_+ + \Delta\theta_-. \quad (10)$$

As seen from Eqs. 5–10, the opening angle decreases (increases) for $p_- > p_+$ ($p_- < p_+$), making the mass shift more negative (positive). However, it will be shown in Sect. 4 that taking into account the opening angle change derived above does not make significant difference in the mass shift. But, the assumption of large kaon momenta does not always hold in the fireball rest frame. However, at low kaon momentum, the change in the opening angle makes larger contribution to the mass shift than derived from above Eqs., therefore including the opening angle change does not change the mass shift qualitatively. For the sake of simplicity, we will neglect it in the following analysis.

3 Monte Carlo Procedure

As mentioned above, the ϕ mass shift due to coulomb effect is positive on average if all K^+K^- pairs are accepted and reconstructed. However, the E859 spectrometer has a limited acceptance in which systematic decrease of the ϕ mass was reported. In order to implement the acceptance, we used a *Monte Carlo* approach.

In the *Monte Carlo* analysis, we generate ϕ 's according to exponential distribution in m_{\perp} with inverse slope 180 MeV, and Gaussian distribution in rapidity centered at 1.2 and with width $\sigma = 0.5$ [3]. The rapidity of the charge fireball in central Si+Au collision is $y_{fb} = 1.13$, which is distinguished from the center of mass rapidity, $y_{cm} = 0.97$. The azimuthal angle Φ of the ϕ meson is chosen randomly, and such that the spectrometer is around $\Phi = 0$. We assume all ϕ 's are created at the origin of the fireball, and decay according to $e^{-\frac{m}{p} \cdot \frac{r}{c\tau}}$, where m , p and $c\tau$ are rest mass, momentum

in the fireball rest frame and decay constant of ϕ . The decay angles, (Θ_K, Φ_K) for K^+ and $(\pi - \Theta_K, \pi + \Phi_K)$ for K^- , are selected isotropically in the ϕ rest frame.

In order to make the Lorenz transformation easy from the ϕ rest frame, (x', y', z') , to the fireball rest frame, (x, y, z) , we select $\Theta_K = 0$ to be the direction of the ϕ momentum and x' -axis to be on the plane of the ϕ motion and the z -axis (thus y' -axis is parallel to the $x - y$ plane). Given the kaon 4-momentum in the ϕ rest frame,

$$p_\pm' = \begin{pmatrix} \pm p_K \sin \Theta_K \cos \Phi_K \\ \pm p_K \sin \Theta_K \sin \Phi_K \\ \pm p_K \cos \Theta_K \\ \sqrt{p_K^2 + m_K^2} \end{pmatrix},$$

where p_K is the ϕ decay momentum and m_K is the kaon rest mass, the kaon 4-momentum in the fireball rest frame can be obtained by

$$p_\pm^\mu = \begin{pmatrix} \cos \Theta \cos \Phi & -\sin \Phi & \sin \Theta \cos \Phi & 0 \\ \cos \Theta \sin \Phi & \cos \Phi & \sin \Theta \sin \Phi & 0 \\ -\sin \Theta & 0 & \cos \Theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & \beta \gamma \\ 0 & 0 & \beta \gamma & \gamma \end{pmatrix} p_\pm' \mu, \quad (11)$$

where β and γ are the velocity and Lorentz factor of the ϕ in the fireball rest frame.

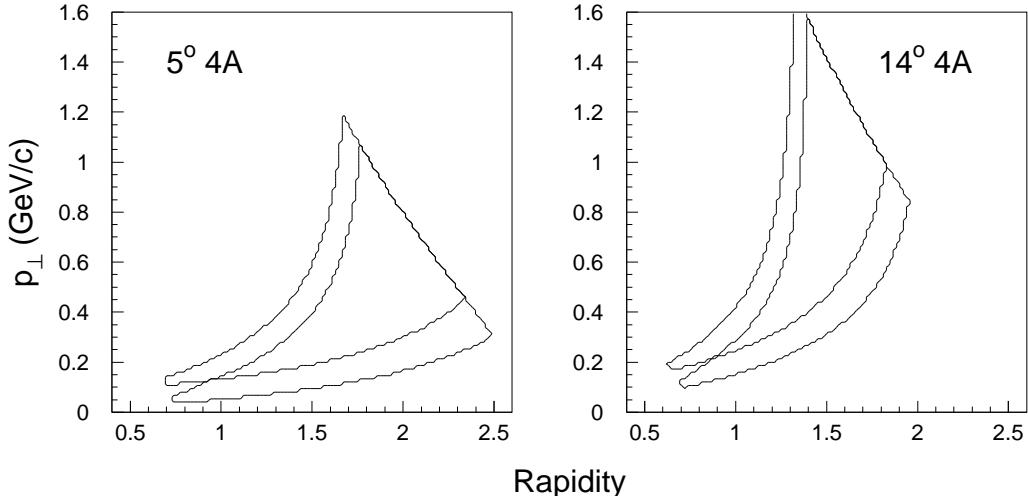


Figure 2: Acceptance of the E859 spectrometer at 5° (left panel) and 14° (right panel) with magnetic field 4A for K^+ (right contour in each plot) and K^- (left contour in each plot)

We then give a \pm coulomb energy shift into K^\pm energy, and calculate their momenta which are measured in the spectrometer. Since the ϕ was measured by E859

with the spectrometer at 5° and 14° and magnetic field of 4A (-4 KG) and 4B ($+4$ KG) [3], we apply only the acceptances of these settings to the resulting kaons. In addition, we require momenta of both kaons to be in the range of $0.4 < p < 3.5$ GeV/c in which the kaons are identified in the experiment [3]. We also let the kaons decay according to $e^{-\frac{m}{p} \cdot \frac{r}{c\tau}}$, where m , p and $c\tau$ are rest mass, momentum in lab system and decay constant of kaon. In the analysis, then, we use only the ϕ 's whose decay kaons are in the acceptance, within the momentum range and do not decay in the spectrometer before the Time-of-Flight wall which locates at 650 cm from the target and was used in kaon identification. Fig. 2 shows the acceptances of K^+ and K^- for magnetic field 4A and spectrometer angles 5° and 14° . The acceptances of K^+ and K^- for magnetic field 4B are identical to those of K^- and K^+ for 4A at the corresponding spectrometer angles.

4 Results

We generated 20 million ϕ 's within rapidity and p_\perp range of $0.5 < y < 2.5$ and $0.2 < p_\perp < 2.2$ GeV/c, respectively. Out of these, about 55 K, 46 K, 10 K and 7.5 K ϕ 's are accepted by the spectrometer for each magnetic field polarity at 5° 2 KG, 5°

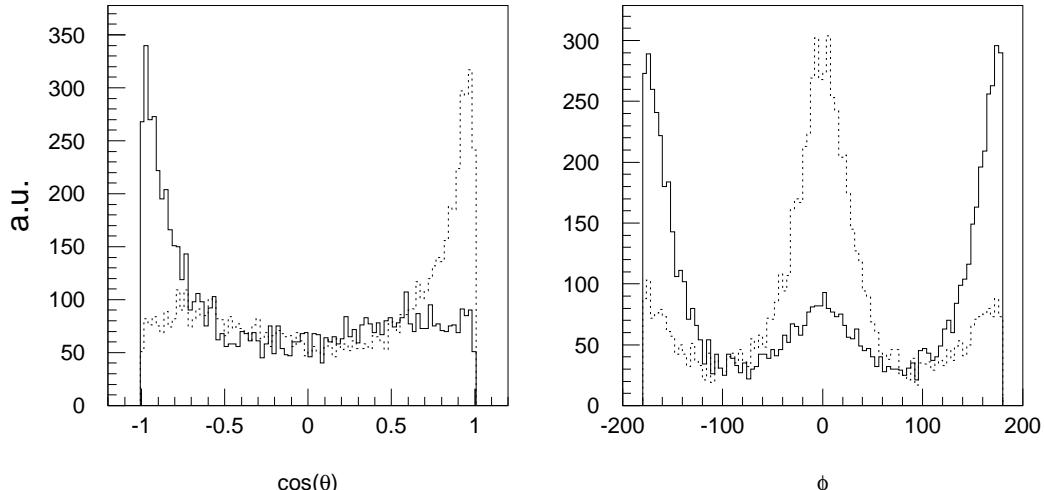


Figure 3: Decay angles of K^+ from ϕ 's whose secondary K^+K^- pairs are accepted and identified in the spectrometer at 14° with magnetic field 4A (solid) and 4B (dotted)

4 KG, 14° 2 KG and 14° 4 KG settings, respectively. As stated in Sect. 3, all ϕ 's decay isotropically in its rest frame. However, decay angles selected by the spectrometer are strongly biased. Fig. 3 shows the decay angles of the ϕ 's whose secondary kaons are accepted in the spectrometer.

Because of the acceptance bias, more K^+K^- pairs are populated in the negative (positive) ϕ mass shift region for A (B) polarity setting of the magnetic field. Fig. 4 shows the contour plots of the accepted K^+K^- pairs in the spectrometer at 14° and with magnetic field 4A and 4B versus K^+ and K^- momenta in the fireball rest frame. By comparing them to Fig. 1, one can see that there will be negative shift in the ϕ invariant mass for the 4A setting, and positive shift for the 4B setting. Profiles of the mass shift as a function of the ϕp_\perp are shown in Fig. 5 for 4A and 4B magnetic field settings with the spectrometer at 5° and 14° .

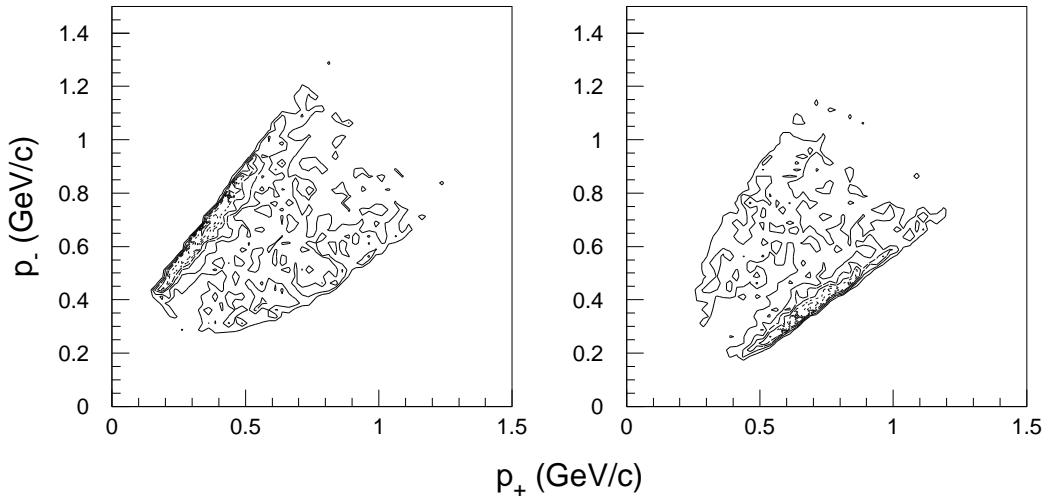


Figure 4: Contour plots of K^+ and K^- momenta of accepted pairs from ϕ decays in the spectrometer at 14° with magnetic field 4A (left) and 4B (right)

One might think that the positive mass shift for magnetic field B polarity should be larger in magnitude than the negative shift for A polarity, because one could just flip the K^+ and K^- decay angles in the ϕ rest frame to have the kaons in the acceptance of the opposite polarity, and because the average mass shift should be positive. However, this is not true because flipping the magnetic field polarity cannot flip the coulomb potential of the positive fireball on K^+ or K^- . Technically, the

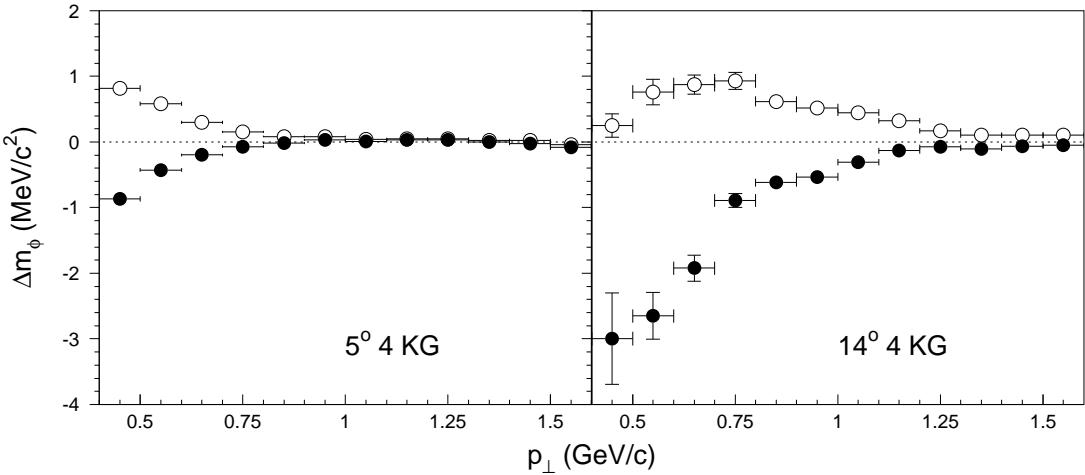


Figure 5: Average ϕ mass shift versus p_{\perp} with spectrometer at 5° and 14° and magnetic field settings of 4A (filled-in circles) and 4B (open circles)

argument goes as follows. The spectrometer does not accept equally a K^+K^- pair in A setting and its image pair (with kaon momenta flipped) in B setting. Consider a kaon pair from a ϕ decay with momenta of $(p_+, p_-) = (p_1, p_2)$ in the fireball rest frame, where $p_1 < p_2$. The mass shift for this pair will be negative and the momenta in lab system will be $(p_+, p_-)_{lab} = (p_1 + \Delta p_1, p_2 - \Delta p_2)$. Consider another kaon pair from a ϕ decay with momenta of $(p_+, p_-) = (p_2, p_1)$ in the fireball rest frame. The mass shift for this pair will be positive and the momenta in lab system will be $(p_+, p_-)_{lab} = (p_2 + \Delta' p_2, p_1 - \Delta' p_1)$. Because the spectrometer has the triangle-shape acceptance in (y, p_{\perp}) as shown in Fig. 2, it has smaller acceptance for lower momentum particle. Thus, the probability for the second pair to be accepted is smaller than that for the first pair at both polarities, therefore the average mass shift combining both polarities is negative. However, the effect becomes smaller when the kaon momenta are large, or the ϕ has large p_{\perp} . This is reflected in the mass shift at large p_{\perp} at which the positive shift is indeed larger than the negative shift in magnitude.

Although the average ϕ mass shift shown in Fig. 5 is large at 14° 4A setting, the mass shift distribution shows an asymmetric peak at a negative value close to zero. Fig. 6 shows the mass shift distributions with different cuts on p_{\perp} of the ϕ . In addition to the ϕ 's described earlier, the figure also includes ϕ 's accepted in the spectrometer out of 10 million ϕ 's that were generated within a more restrictive p_{\perp}

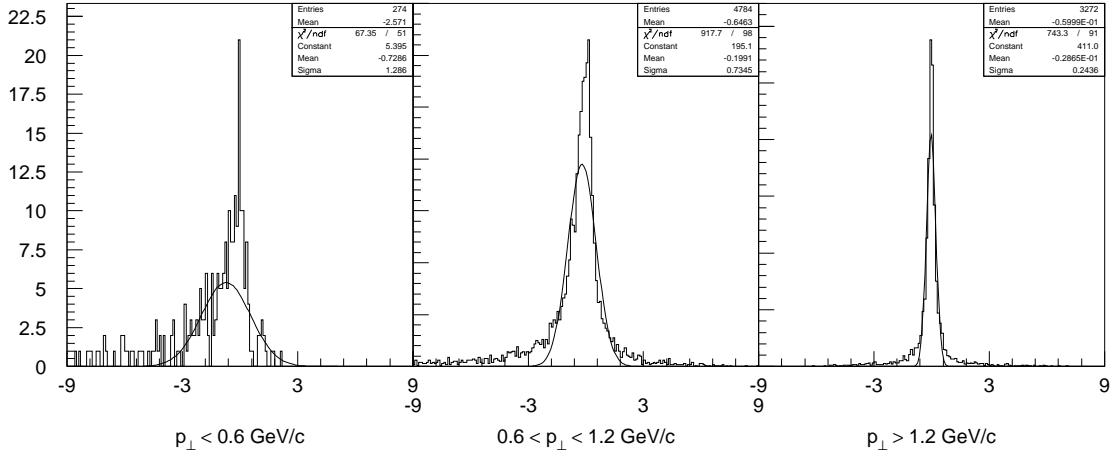


Figure 6: Distributions of ϕ mass shift (MeV/c^2) in three p_\perp bins

range of $0.2 < p_\perp < 0.8 \text{ GeV}/c$. Gaussian fits to the distributions in the range of $(-5, 5) \text{ MeV}/c^2$ indicate that the Gaussian means are significantly smaller than the averages.

Including the change in the opening angle, one will get enhanced mass shift as discussed in Sect. 2. Fig. 7 shows the mass shift with the angle change included versus that without. The effect is not significant. Although the assumption of high kaon momenta in the fireball rest frame used in the calculation of the opening angle

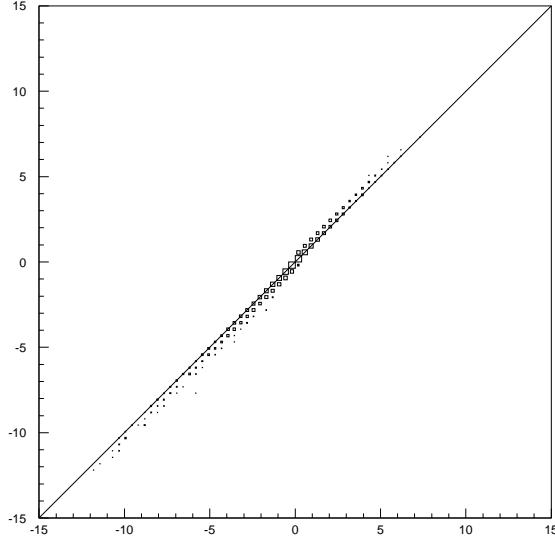


Figure 7: ϕ mass shift (MeV/c^2) with opening angle change included versus that without

change does not always hold, including the opening angle change always enhance the mass shift, therefore does not change the conclusion qualitatively.

Since coulomb potential is proportional to Z , the ϕ mass shift increases with collision centrality (impact parameter). However, the increase is not significant once the Si+Au collision reaches a certain centrality.

5 Limitations of the Analysis

There are many unphysical assumptions in the analysis presented, preventing us from drawing quantitative conclusions. In the real world, the positive charges in the fireball are not static or confined in a spherical volume, and the charge density is not uniform. As many experimental data suggest, the fireball is expanding in both longitudinal and transverse directions. Therefore, the secondary kaons from ϕ decays might co-move with the charges longitudinally. Besides, it is not all clear whether or not the kaons from ϕ decays are fast enough to escape quickly from the fireball so that they are less effected by the transverse expansion of the fireball. To take into some of the co-moving effect between the charges in the fireball and the decay kaons, we alternatively apply an cylinder fireball with uniform charge density to the same analysis, so that longitudinal momenta of the kaons do not change. We assume a cylinder source with radius of that of the Si projectile, R , and with length of the diameter of the Au target, D , and approximate the coulomb potential for a positive unit charge as

$$V(r) = \begin{cases} \frac{Z\alpha}{r} & (r \geq D/2) \\ \frac{Z\alpha}{D/2} \left(1 + \ln \frac{D/2}{r}\right) & (R \leq r < D/2) \\ \frac{Z\alpha}{D/2} \left\{ \frac{1}{2} \left[3 - \left(\frac{r}{R}\right)^2 \right] + \ln \frac{D/2}{r} \right\} & (r < R) \end{cases}, \quad (12)$$

where r is in cylinder coordinates. The ϕ mass shift thus obtained are more pronounced (by factor of 1.5–2) than those shown in Sect. 4 for a spherical uniform charge fireball.

In the calculation presented in the previous sections, we have neglected coulomb interaction between the secondary kaons and the target fragments. However, we can estimate the effect by treating the target fragments as the source of the net positive

charge, thus effectively assuming all the ϕ 's are produced at $t = 0$. The mass shift thus obtained is consistent with zero and has much less p_\perp dependence. We note that this is the upper limit of the effect because any ϕ 's produced at later times feel less coulomb potential from the target fragments. Although the target fragments have to be considered together with the fireball in a realistic calculation, they are not expected to make a significant contribution compared to that from the fireball alone.

6 Prediction on Au+Au Collision

We applied the same analysis to central Au+Au collision at 11.1 A·GeV/c. In this case, net charge and radius of the created fireball are $Z = 158$ and $R = 8.2$ fm, respectively. The fireball rapidity is $y_{fb} = 1.6$, and is as same as the center of mass rapidity. The ϕ 's were generated with m_\perp inverse slope 180 MeV, and Gaussian distribution in rapidity centered at $y_{fb} = 1.6$ and with width $\sigma = 0.5$. The average ϕ mass shift as a function of p_\perp is shown in Fig. 8 for spectrometer at 14° and magnetic field 4A and 4B. As expected, the shifts are about 2 times of those in central Si+Au collision for the same settings.

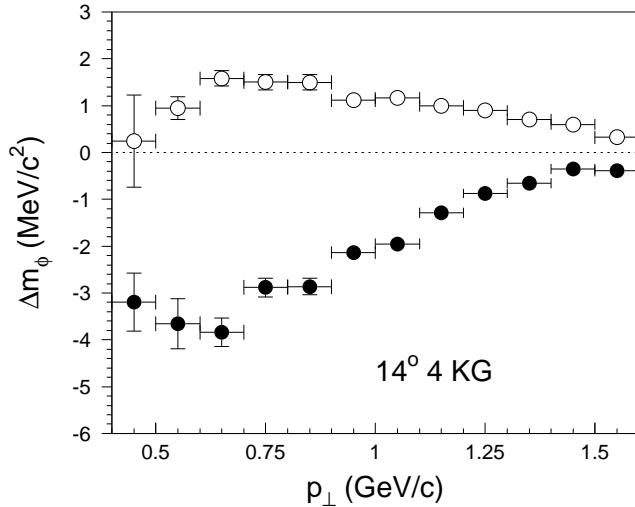


Figure 8: Average ϕ mass shift versus p_\perp with spectrometer at 14° and magnetic field 4A (filled-in circles) and 4B (open circles)

In order to answer whether or not ϕ mass shift, if any, in central Au+Au collision

is due to non-conventional effect of chiral symmetry restoration at extreme conditions, one has to study differences in the data taken with various spectrometer and magnetic field settings. It should be pointed out that the study of the ϕ mass shift due to coulomb effect presented here does not dilute any interest in the subject of ϕ mass shift from chiral symmetry restoration. If the chiral symmetry is restored in central Au+Au collision, anomalously large drop in the ϕ mass is expected and should be able to manifest itself from coulomb effect discussed here.

7 Summary and Outlook

In summary, we have reported here an analysis of ϕ mass shift due to coulomb interaction between ϕ decay kaons and a positive charge fireball. The shift is negative (positive) on first order of $Z\alpha$ when the K^- momentum is larger (smaller) than the K^+ 's in the fireball rest frame. Average ϕ mass shift, summing over all possibilities of decay kinematics, is slightly positive due to second order effect. We implemented the E859 spectrometer acceptance in our calculation for settings at which systematic decrease in the ϕ invariant mass was observed [4], and found that the shift caused by coulomb effect is in semi-quantitative agreement with the experimental observation. We demonstrated that the shift resulted from strongly biased ϕ decay angles selected by the E859 spectrometer. We applied the same analysis to central Au+Au collision, and predicted that the shift from coulomb effect is 2 times of that in central Si+Au collision.

Due to many limitations and classic nature of the analysis, the paper offers only semi-quantitative look at coulomb effect on the ϕ invariant mass. To fully understand the effect, further analysis is clearly needed. For example, one can extend the quantum relativistic analysis of coulomb effect on single particle spectra in [5] to kaon pairs from ϕ decays, thus analyzing the ϕ mass shift in the quantum relativistic framework.

8 Acknowledgments

The author is indebted to W. A. Zajc, B. A. Cole, S. Nagamiya, M. D. Moulson and Y. Wu for valuable comments and discussions. This work was supported by the U. S. Department of Energy under contract DE-FG02-86-ER40281 with Columbia University.

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